Expectation and Thin Value in No-limit Hold 'em: Profit comes with Variance

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People get confused in a number of ways about betting thinly for value in NLHE cash games. It is simplest to consider a river value bet. When Hero bets and is called by her opponent (Villain), if she wins more than half of the time, the wager will be profitable. Winning 50% of the time is the break even point for river thin value betting when your opponent can only call, but what does that number really mean in the context of the Villain's holdings? People are often uncertain about how often they can expect to get called by better hands and deciding how much to bet. Below, a realistic example is analyzed in detail so the elements in deciding whether, when, and how much to bet are evident. The example also makes clear that proficient thin value betting will increase both a bettor's (Hero's) win rate and variance significantly.

It is often necessary to bet small on the river to ensure that the hands that are inferior to Hero's may call, thus tuning the bet size to establish a weaker calling range. Such scenarios make even analyzing river bets somewhat complicated. Therefore, an example is presented where there are an equal number of combinations of hands in Villain 's range that are better and worse than Hero 's independent of river bet sizing.

Consider $2/$5 NLHE, $600 stacks (120 big blinds) for both Hero and Villain.

Hero raises $20 from middle position with A[club]Q[heart] and is called by a Villain who is a regular recreational player. The blinds also call for a total of three callers (this is a typical loose passive live $2/$5 NLHE game).

Total pot: $80

Flop: A[spade]7[club]2[spade]

Hero bets $60 on flop and only the Villain calls.

Total pot: $200

Turn: A[spade]7[club]2[spade]A[diamond]

Hero bets $120 on turn and Villain calls.

Total pot: $440

River: A[spade]7[club]2[spade]A[diamond]6[heart]

Hero and Villain both have $400 remaining.

What should Hero do on the river? Suppose Hero knows the Villain well, she is relatively tight and always has AK, AQ or AJ after calling the flop and turn bets. Villain would have folded a flush draw on the paired turn and raised a set for protection on the flop. These are the only aces the Villain calls a bet with preflop vs. Hero; there are a significant number of regulars with this profile at $2/$5 NLHE. Whatever reasonable bet we make, the relatively passive Villain will just call with these hands. The combinations of Villain‘s hands are as follows:

AK: 4 combinations, one Ace and four Kings possible
AQ: 3 combinations, one Ace and three Queens possible
AJ: 4 combinations, one Ace and four Jacks possible

Suppose we go all in for the remaining $400. Since we beat 4 combinations (AJ), tie 3 (AQ) and lose to 4 combinations (AK), Hero‘s expected value is exactly zero on the river bet. Indeed, it is zero no matter what sizing we choose. So, it doesn’t matter what amount is bet except to consider variance – a bigger bet creates larger fluctuations in Hero‘s bankroll.

However, let’s now assume that the somewhat passive Villain would raise us preflop with AK 20% of the time. Because there was no preflop reraise, it is now slightly more likely that the Villain has either AQ or AJ than AK (the kind Bayesian inference one should make at the table). Now it is more profitable to bet as large as possible on the river, even though the number of combinations of hands ahead and behind of Hero‘s are the same, and the Villain’s calling frequency is 100% without respect to bet size.

This is the essence of value betting on the river. When called, Hero needs to have the best hand at least 50% of the time for zero or greater expected value on the even money bet. If this condition is met, Hero chooses a bet size that maximizes the expected value. To be precise, each bet size would require separate consideration. Sometimes Hero needs to consider that a larger bet will only get called by a stronger part of the Villain’s range. Further, if it is possible to establish Villain’s range for calling a particular size, first Hero needs to count combinations ahead and behind of her hand. Next, the likelihood of Villain having each combination is established and then a weighted average gives the expected value of the bet. For example, because Villain will raise preflop 20% of the time, the AK holding needs to be discounted. This can be assessed in terms of the combinations of holdings the Villain has on the river:

AK: 0.8 *4=3.2 combinations, one Ace and four Kings possible; this represents the 80% frequency of Villain just calling preflop
AQ: 3 combinations, one Ace and three Queens possible
AJ: 4 combinations, one Ace and four Jacks possible

Now consider both a $200 and $400, all in, river bet under this scenario. For a $200 river bet the expected value is:

River Think Value Bet

Hero wins 4 times, ties 3 and loses 3.2:
Win: 4 / (3.2+3+4) * $200 = +$78.43
Tie: 3 / 10.2 = $0.00
Lose: 3.2 / 10.2 * $200 = -$62.74

Expected value for $200 river bet: +$15.69
Expected value for $400 river bet: +$31.37

Villain wins 3.2 times, ties 3 and loses 4:
Win: 3.2/ (3.2+3+4) *$200 = +$58.62
Tie: 3/ 10.2 = $0.00
Lose: 4/ 10.2 *$200 = -$78.43

Expected value for $200 river bet: -$15.69
Expected value for $400 river bet: -$31.37

Had Hero bet $400, replace the $200 to get double the expected value for our river bet of $31.37. One multiplies the frequency (for example, the number of winning combinations, 3.2, divided by total number, 10.2) by the bet to get the expected value. Note, that while the Hero's river bet is even money (independent of the prior action), the Villain's considerations are different, in that raising, calling or folding are directly influenced by the odds offered by both Hero's bet and the existing pot size. Here, the Villain always calls with a fixed range and the expected values are equal and opposite.

How much does Hero benefit from being a thin value bettor? Where does profit come from in NLHE? Let's consider the expected value for the entire hand assuming a $400 all in bet on the river – after all this maximizes both the Hero's win rate and variance. First, note, when the villain does not raise preflop, the Hero and Villain have both put ($20+$60+$120+$400=) $600 into the pot and there is $40 from players who folded. However, we now have to account for the AK that the Villain raises preflop (where everyone folds, and in this case the blinds only contribute $2 and $5 respectively).

Look at things this way: Hero always has AQ, while Villain has AJ (4 times), AQ (3 times), AK calling (3.2 times) and AK raising (0.8 times) – for a total of 11 opposing combinations of hands. (Note, only the portion of Villain's range that plays the entire hand in the prescribed fashion is considered and not, for example, hands that might call the flop, such as K[spade]Q[spade] and 77, but would proceed differently.) When Hero is called preflop, the final pot is $1,240. This happens 10.2/11*100 = 92.7% of the time. When raised preflop, Hero folds to the tight regular player. This happens 0.8/11 * 100 = 7.3% of the time. When folding, Hero loses $20 to the Villain. When the dust settles, the accounting is as follows.

Both Realizations of the Entire Hand

When Villain calls preflop both players put their entire $600 stack in the pot and the other callers put in $40 and fold. When Villain raises preflop, Hero loses $20 and the blinds lose $7.

Hero (AQ)

Four Wins (Villain has AJ):
4 / (3.2+3+4) *($40+$600+$600-$600) = +$250.98

Three Ties (Villain has AQ):
3/ 10.2*($1240 /2 -$600) = +$5.88

Four Losses (Villain has AK)
Loses 3.2 combinations (Villain calls with AK preflop):
3.2/ 10.2* $600= -$188.24

Loses 0.8 combinations (Villain raises AK preflop):
0.8: = -$20

Hero's expected value from the hand:
+$68.63 * (10.2/11) - $20 *(0.8/11) = +$62.18

Villain (AK, AQ, AJ)

Four Wins with AK
Wins 3.2 combinations calling with AK
3.2 / (3.2+3+4) ** ($1240-$600) = +$200.78

Wins 0.8 combos raising with AK
0.8: = +27. ($20 from Hero, $7 and from blinds)

Three Ties calling with AQ
3/ (3.2+3+4) *($1240 /2 -$600) = +$5.88

Four Losses calling with AJ
4/ (3.2+3+4) * $600= -$235.29

Villain's expected value from the hand:
-$28.62 * (10.2/11) + $27 *(0.8/11) = -$24.58

What have we learned? In constructing thin river value bets, Hero assesses what the Villain's calling range is for a given bet size, counts the combinations of hands ahead and behind her hand (weighted by the relative frequency of having the hand) and decides whether to bet. The bet that gives the highest expected value is the optimal choice.

This analysis shows why it is difficult to say how often one should "value own" themselves (getting called by better when betting for value) to achieve maximum expected value. For example, if Hero bets larger, she will get called by stronger hands but wins more when her hand is best. Notice that the river bet, while showing a meaningful positive expectation, also leads to large variance; 39% of the time we win an extra $400 but we break even or lose $400 most of the time. This demonstrates that one must be properly bankrolled and willing to suffer the variance in order to obtain maximum value.

Also, note how thin our value bet is on the river. It all depends on the Villain's slightly depleted AK holding that occasionally reraises preflop and causes us to fold, losing our initial $20 investment. But, the extra roughly thirty dollars profit from our river bet – a result of correctly assessing our opponents range and acting accordingly – represents a reasonable hourly rate for a mid-stakes live professional player, and is thus critical.

The down side to playing so well is that about a third of the time Hero loses an extra eighty big blinds, increasing variance significantly. This is a general principle in NLHE and poker more generally. As one improves and exploits thinner edges, expected value goes up (modestly) and variance increases (steeply) – the math makes it so. (This was first shown in *Gambling Theory and Other Topics* by Mason Malmuth) That is what exploiting small edges like value betting thinly means. So, maintain an adequate bankroll and bet as thinly as is profitable. Put another way, with a limited bankroll, an expert who exploits these types of bets, as opposed to a merely good player who does not, is more likely to go broke even though his win rate is higher.

Lastly, even good players also often misinterpret this spot when evaluating the associated poker session. Suppose Hero takes the entire pot – how much did they really win? In actual dollars quite a bit, taking home an additional $640. However, the expected value was actually quite small and the win is simply part of variance – this means to expect sessions with a big loss to even most of this out if Hero plays a similarly effective style consistently. Thin value giveth and taketh away. Or put another way, when considering Sklansky bucks (the expected value in the hand against each of Villain's holdings), Hero's hand is either way ahead, tied or way behind at all points. But given that Hero will play the hand the same way against Villain's range (hands that they will play this way against Hero), they are not the most salient metric in this case. Instead, consider that Galfond bucks, Hero's actual hand against the Villain's range, give an expected value of a few big blinds *vs.* the Villain (a lot of the profit in the example comes from the other players who called preflop and folded the flop). In this example, Hero is quite likely to play this hand similarly against comparable Villains in the future, making Hero's profit modest. This is not an academic result – while his profit may be large since he won the pot, Hero's expectations, while positive, is much smaller.

In sum, when value betting thinly, establish the Villain's range with respect to your possible betting sizes. Consider the hands the Villain could call with, ahead and behind, and how often she would have these holdings. If there is a sizing (if it is small, consider you might get bluff raised occasionally) where half Villain's combinations, weighted by frequencies of playing them as the action unfolded, are behind, bet. Pick the sizing that

maximizes the expected value against this filtered range – for example, if you bet too large Villain might fold everything. That's it. If poker were easy everyone would play profitably.